Geometric Algorithms: Convex Hull in Plane

Introduction

The first scene is just an illustration of a convex hull of a finite set of points in the plane.

Input Set

The input for the problem is a finite set of points in the plane. This scene shows how a user can manipulate with the set. The menu in the control bar allows for four modes of using the mouse:

- *Mouse inactive* - no mouse action is possible.
- *Insert site* - a mouse click inserts a new element of the input set (unless an already existing site has been clicked).
- *Delete site* - a site can be deleted by a mouse click.
- *Move site* - a mouse-down over a site catches it and the site can be dragged to another location.

Another possibility is (after possible change of the field that determines the number of the input set elements) clidking the button New Input to create a new input set of the given size.

Convex Hull

Modify the set of sites as it has been explained in the previous scene - this scene shows always the convex hull of the set. Moving a site around might be found funny.

Orientation

The boundary of the convex hull od a finite planar set is a polygon. The boundary of the polygon is a simple closed curve that can be oriented. After checking Show Orientation you will see the boundary oriented clockwise.

Incremental Building

This scene shows the main idea of the convex hull algorithm. The algorithm is *incremental*: using the sites ordered left-to-right by their x-coordinates, the algorithm first creates a convex hull of the three leftmost sites - a triangle. Then, each click of the button Step extends the hull by adding one more site (in the order of increasing x-coordinates).

Computation

This section shows step-by-step how the incremental building of the convex hull can be implemented. It is assumed that the sites are already sorted by their x-coordinates. The first step, building a triangle of 3 leftmost sites is easy. Then, for each new site, the following is repeated, until the hull is found:

1. denote the rightmost site of the present hull by A;
2. go clockwise from A along the perimeter of the hull and denote by B and C the first two sites you meet;
3. determine the size of the angle with the vertex in B and branches BA and BC;
4. if the angle is smaller than 180 degrees (marked by red color), then add the triangle ABC to the hull and go to 2; otherwise (the angle is blue), proceed to 5;
5. go counter-clockwise from A along the perimeter of the hull and denote by B and C the first two sites you meet;
6. determine the size of the angle with the vertex in B and branches BA and BC;
7. if the angle is smaller than 180 degrees (marked by red color), then add the triangle ABC to the hull and go to 5; otherwise (the angle is blue), finish extending the hull by the site A;

Data Structure

Complex Insert

Complexity